

## Linear approximation and higher order partial derivatives

1. The monthly cost function for ACME Widgets is

$$C = 0.02Q_A^2 + 0.01Q_AQ_B + 0.03Q_B^2 + 35Q_A + 28Q_B + 5000,$$

where  $Q_A$  and  $Q_B$  are the monthly outputs of type A widgets and type B widgets, respectively, measured in 100s of widgets. The cost is measured in dollars.

- a. Compute the marginal cost of type A widgets and the marginal cost of type B widgets, if the monthly outputs are 25000 type A widgets and 36000 type B widgets.
- b. Suppose that production of type A widgets is held fixed at 25000, and production of type B widgets is increased from 36000 to 36050. Use your answer to part a. to estimate the change in cost to the firm.
- c. Suppose that production of type A widgets is increased from 25000 to 25060, and production of type B widgets is increased from 36000 to 36040. Use your answer to part a. to estimate the change in cost to the firm.

2. The demand function for a firm's product is given by  $Q = \frac{30\sqrt{6Y + 5p_s}}{3p + 5}$ , where

- $Q$  is the monthly demand for the firm's product, measured in 1000's of units,
  - $Y$  is the average monthly disposable income in the market for the firm's product, measured in 1000s of dollars,
  - $p_s$  is the average price of a substitute for the firm's product, measured in dollars,
  - $p$  is the price of the firm's product, also measured in dollars.
- a. Find  $Q$ ,  $Q_Y$ ,  $Q_{p_s}$  and  $Q_p$  when the monthly income is \$2500 and the prices are  $p_s = 17$  and  $p = 15$ . Round your (final) answers to two decimal places.
  - b. Compute the *income-elasticity of demand* for the firm's product at the point in part a.
  - c. Use *linear approximation* and your answer to a. to estimate the change in demand for the firm's product if the price of the firm's product increases to \$16 and the price of substitutes increases to \$18, but income remains fixed.
  - d. Use your answer to part b. to estimate the *percentage* change in demand for the firm's product if the average income increases to \$2600 while the prices stay the same as they were in part a.

3. Find the indicated partial derivatives of the functions below.

(a)  $z = 3x^2 + 4xy - 5y^2 - 4x + 7y - 2,$

$$z_{yx} =$$

$$z_{xx} =$$

(b)  $F(u, v, w) = 60u^{2/3}v^{1/6}w^{1/2}$

$$\frac{\partial^2 F}{\partial w \partial u} =$$

$$\frac{\partial^2 F}{\partial v^2} =$$

(c)  $w = x^2 z \ln(y^2 + z^3)$

$$w_{xx} =$$

$$w_{yz} =$$

$$w_{xyz} =$$

(d)  $q(u, v) = \frac{u^2 v - 3uv^3}{2u + 3v}$

$$\frac{\partial^2 q}{\partial u^2} =$$