

## Solutions

1. Find the indicated partial derivatives of the functions below.

(a)  $f(x, y, z) = 2x^3y^2z + 3x^2y^5z^3 - 4xy^3 + yz^4$ .

$$f_x = 6x^2y^2z + 6xy^5z^3 - 4y^3$$

$$f_y = 4x^3yz + 15x^2y^4z^3 - 12xy^2 + z^4$$

$$f_z = 2x^3y^2 + 9x^2y^5z^2 + 4yz^3$$

(b)  $w = \frac{u^2v}{u + v^2}$ .

$$\frac{\partial w}{\partial u} = \frac{2uv(u + v^2) - u^2v \cdot 1}{(u + v^2)^2} = \frac{u^2v + 2uv^3}{(u + v^2)^2}$$

$$\frac{\partial w}{\partial v} = \frac{u^2(u + v^2) - u^2v(2v)}{(u + v^2)^2} = \frac{u^3 - u^2v^2}{(u + v^2)^2}$$

(c)  $F(x, y, z, \lambda) = 10 \ln(x^2y^5z^3) - \lambda(5x + 2y + 8z)$ .

$$F_x = \frac{10}{x^2y^5z^3} \cdot 2xy^5z^3 - 5\lambda = \frac{20}{x} - 5\lambda$$

If you use properties of the log function to simplify  $F$  before you differentiate, the differentiation is a little easier:

$$F(x, y, z, \lambda) = 20 \ln x + 50 \ln y + 30 \ln z - \lambda(5x + 2y + 8z) \implies F_x = \frac{20}{x} - 5\lambda$$

$$F_\lambda = -(5x + 2y + 8z).$$

(d)  $z = 2y^2e^{x^2y}$ .

$$\frac{\partial z}{\partial x} = 2y^2e^{x^2y} \cdot 2xy = 4xy^3e^{x^2y} \quad \text{In this one, } y^2 \text{ behaves like a constant factor.}$$

$$\frac{\partial z}{\partial y} = 4ye^{x^2y} + 2y^2e^{x^2y} \cdot x^2 = (4y + 2x^2y^2)e^{x^2y}. \quad \text{We have to use the product rule here.}$$

(e)  $z = 3x^2 + 4xy - 5y^2 - 4x + 7y - 2$ ,

$$z_x = 6x + 4y - 4$$

$$z_{yx} = z_{xy} = 4$$

(f)  $F(u, v, w) = 60u^{2/3}v^{1/6}w^{1/2}$

$$\frac{\partial F}{\partial u} = 40u^{-1/3}v^{1/6}w^{1/2}$$

$$\frac{\partial F}{\partial w} = 30u^{2/3}v^{1/6}w^{-1/2}$$

(g)  $w = x^2z \ln(y^2 + z^3)$

$$w_x = 2xz \ln(y^2 + z^3)$$

$$w_y = x^2 z \cdot \frac{2y}{y^2 + z^3} = \frac{2x^2 yz}{y^2 + z^3}$$

$$w_z = x^2 \ln(y^2 + z^3) + x^2 z \cdot \frac{3z^2}{y^2 + z^3} = x^2 \ln(y^2 + z^3) + \frac{3x^2 z^3}{y^2 + z^3}$$

$$(h) \quad q(u, v) = \frac{u^2 v - 3uv^3}{2u + 3v}$$

$$\frac{\partial q}{\partial u} = \frac{(2uv - 3v^3)(2u + 3v) - 2(u^2 v - 3uv^3)}{(2u + 3v)^2} = \frac{2u^2 v + 6uv^2 - 9v^4}{(2u + 3v)^2}$$

$$\frac{\partial q}{\partial v} = \frac{(u^2 - 9uv^2)(2u + 3v) - 3(u^2 v - 3uv^3)}{(2u + 3v)^2} = \frac{2u^3 - 18u^2 v^2 - 18uv^3}{(2u + 3v)^2}$$