## More definite integrals and more applications

1. What are the Producers' surplus and Consumers' surplus for the market with supply function

$$
p=0.05 q^{2}+3 q+5
$$

and demand function

$$
p=100-0.75 q
$$

2. Find the average value of the function $f(x)=\frac{x^{4}-1}{x^{2}}$ on the interval $[1,3]$.
3. Use the table of integral formulas in Appendix B in the textbook to help compute the integrals below.
(a) $\int \frac{4 d x}{5 x \sqrt{x^{2}+9}}=$
(d) $\int_{0}^{3} \frac{2 d v}{\sqrt{v^{2}+16}}=$
(b) $\int \frac{2 e^{2 x} d x}{\sqrt{9+4 e^{x}}}=$
(e) $\int 5 x^{3} \ln x d x=$
(c) $\int_{0}^{10} 200 t^{2} e^{-0.06 t} d t=$
(f) $\int_{0}^{2} \frac{3+5 x}{2+7 x} d x=$

Hints: (b) Start with the substitution $u=e^{x}$ (and remember that $\left.e^{2 x}=\left(e^{x}\right)^{2}\right)$.
(f) Write the integrand as a sum of two simpler terms.
4. Compute the present value of the continuous annuity that pays at the continuous rate $f(t)=250 t$ for $T=20$ years, where the constant interest rate is $r=4.75 \%$.
5. Let $y=f(x)$ satisfy (i) $\frac{d y}{d x}=3 x y^{2}$ and (ii) $y(1)=2$. Find the function $f(x)$.

