

The definite integral and applications

1. Compute the definite integrals below.

(a) $\int_0^2 2x^3 + x^2 - 5x + 2 \, dx =$

(d) $\int_0^{20} 500e^{-0.04t} \, dt =$

(b) $\int_1^8 2\sqrt[3]{t} + \frac{3}{\sqrt[3]{t^2}} \, dt =$

(e) $\int_0^4 3t\sqrt{t^2 + 9} \, dt$

(c) $\int_0^4 \frac{5}{4x+1} \, dx =$

2. Find the area of the region bounded by the graphs $y = 2\sqrt{x}$ and $y = 1 - 2x$, and the lines $x = 1$ and $x = 4$.
3. Find the Gini coefficient of inequality for the nation with income distribution curve

$$y = 0.5x^3 + 0.3x^2 + 0.2x,$$

where $y \cdot 100\%$ is the percentage of national income earned by the poorest $x \cdot 100\%$ of the population.

4. The marginal propensity to save of a small nation is given by

$$\frac{dS}{dY} = \frac{Y + 5}{9Y + 10},$$

where savings S and national income Y are both measured in billions of dollars. Express the total change in national savings when income increases from \$10 billion to \$15 billion as a definite integral, and find its value. What is the total change in national consumption?