Study Guide 1

Solutions

1. Compute the differentials of the functions below

(a) $y = 3x^2, \quad dy = 6x\,dx$

(b) $u = e^{t^2}, \quad du = 2te^{t^2}\,dt$

(c) $y = \ln(2x^3+1), \quad dy = \dfrac{6x\,dx}{2x^3+1}$

(d) $w = s^2e^{3s}, \quad dw = (2se^{3s} + 3s^2e^{3s})\,ds$

2. Compute the indefinite integrals below.

(a) $\displaystyle\int 3x^3 - 4x^2 + 3x + 2\,dx = \frac{3}{4}x^4 - \frac{4}{3}x^3 + \frac{3}{2}x^2 + 2x + C$

(b) $\displaystyle\int 4\sqrt{t} + \frac{2}{\sqrt[3]{t}}\,dt = \int 4t^{1/2} + 2t^{-1/3}\,dt = \frac{4}{3/2}t^{3/2} + \frac{2}{2/3}t^{2/3} + C = \frac{8}{3}t^{3/2} + 3t^{2/3} + C$

(c) $\displaystyle\int \frac{3x^2+2x-1}{4x^3}\,dx = \int \frac{3}{4}x^{-1} + \frac{1}{2}x^{-2} - \frac{1}{4}x^{-3}\,dx = \frac{3}{4}\ln|x| - \frac{1}{2}x^{-1} + \frac{1}{8}x^{-2} + C$

(d) $\displaystyle\int (x^2+1)(x+3)^2\,dx = \int (x^2+1)(x^2+6x+9)\,dx = \int x^4+6x^3+10x^2+6x+9\,dx$

$$= \frac{1}{5}x^5 + \frac{3}{2}x^4 + \frac{10}{3}x^3 + 3x^2 + 9x + C$$

3. Find the function $y = f(x)$, given that $y' = x - \dfrac{1}{x}$, and $f(1) = 3$.

First, we integrate

$$\int f'(x)\,dx = \int x - \frac{1}{x}\,dx = \frac{x^2}{2} - \ln|x| + C.$$

I.e., $f(x) = \dfrac{x^2}{2} - \ln|x| + C$. Now we use the initial value to solve for C:

$$3 = f(1) = \frac{1^2}{2} - \ln 1 + C = \frac{1}{2} + C \implies C = 3 - \frac{1}{2} = \frac{5}{2}.$$

Conclusion: $f(x) = \dfrac{x^2}{2} - \ln|x| + \dfrac{5}{2}$.

4. The marginal revenue function for a firm is

$$\frac{dr}{dq} = 200 - q^{2/3}.$$

Find the firm's demand function.

The demand function $p = f(q)$ is found by dividing the revenue function, $r(q)$, by q, i.e., $p = r/q$. The revenue function is found by solving the initial value problem, $r' = 200 - q^{2/3}, \quad r(0) = 0$. First,

$$r = \int 200 - q^{2/3}\, dq = 200q - \frac{3}{5}q^{5/3} + C.$$

Next, the initial value $r(0) = 0$ implies that $C = 0$, so $r = 200q - \frac{3}{5}q^{5/3}$, and the demand function is

$$p = \frac{r}{q} = 200 - \frac{3}{5}q^{2/3}.$$

5. A firm's fixed cost is \$12000, and their marginal cost function is

$$\frac{dc}{dq} = (q + 1000)^{1/3} + 50.$$

Find the firm's cost function.

Another initial value problem. First,

$$c = \int (q + 1000)^{1/3} + 50\, dq = \frac{3}{4}(q + 1000)^{4/3} + 50q + K,$$

(using the substitution $u = q + 1000$ to compute the integral).

The initial value is given by $c(0) = 12000$ (because fixed cost $= c(0)$), which we use to solve for the constant of integration, K:

$$12000 = c(0) = \frac{3}{4}1000^{4/3} + K = 7500 + K \implies K = 4500.$$

So the cost function is $c = \dfrac{3}{4}(1000 + q)^{4/3} + 50q + 4500.$