Final Exam

June 11, 2014

Instructions

- Please turn off all phones and other electronic devices.
- There are 6 questions worth a total of 54 points. 100%=50 points.
- No notes or books. A table of integration formulas is provided.
- You ***may*** use a simple scientific calculator. ***No*** graphing or programmable calculators.
- ***Read the questions carefully and check your answers.***
- ***For full credit—show all your work.***

Good Luck!!!

NAME: ____

Problem	Score
1	/8
2	/8
3	/8
4	/10
5	/10
6	/10
Total	/50

Selected Integration Formulas

Basic rules.

1. $\displaystyle\int u^k\,du = \frac{u^{k+1}}{k+1} + C, \quad k \neq -1.$

2. $\displaystyle\int \frac{1}{u}\,du = \ln|u| + C.$

3. $\displaystyle\int e^u\,du = e^u + C.$

4. $\displaystyle\int f(u) \pm g(u)\,du = \int f(u)\,du \pm \int g(u)\,du.$

5. $\displaystyle\int c\cdot f(u)\,du = c\cdot \int f(u)\,du.$

Rational forms containing $(\mathbf{a}+\mathbf{bu})$*.*

6. $\displaystyle\int \frac{du}{a+bu} = \frac{1}{b}\ln|a+bu| + C.$

7. $\displaystyle\int \frac{u\,du}{a+bu} = \frac{u}{b} - \frac{a}{b^2}\ln|a+bu| + C.$

8. $\displaystyle\int \frac{u^2\,du}{a+bu} = \frac{u^2}{2b} - \frac{au}{b^2} + \frac{a^2}{b^3}\ln|a+bu| + C.$

9. $\displaystyle\int \frac{u^2\,du}{(a+bu)^2} = \frac{u}{b^2} - \frac{a^2}{b^3(a+bu)} - \frac{2a}{b^3}\ln|a+bu| + C.$

Forms containing $\sqrt{\mathbf{a}+\mathbf{bu}}$*.*

10. $\displaystyle\int u\sqrt{a+bu}\,du = \frac{2(3bu-2a)(a+bu)^{3/2}}{15b^2} + C.$

11. $\displaystyle\int \frac{u\,du}{\sqrt{a+bu}} = \frac{2(bu-2a)\sqrt{a+bu}}{3b^2} + C.$

12. $\displaystyle\int \frac{u^2\,du}{\sqrt{a+bu}} = \frac{2(3b^2u^2-4abu+8a^2)\sqrt{a+bu}}{15b^3} + C.$

Exponential and logarithmic forms.

13. $\displaystyle\int e^{au}\,du = \frac{e^{au}}{a} + C.$

14. $\displaystyle\int ue^{au}\,du = \frac{e^{au}}{a^2}(au-1) + C.$

15. $\displaystyle\int u^n e^{au}\,du = \frac{u^n e^{au}}{a} - \frac{n}{a}\int u^{n-1}e^{au}\,du\,.$

16. $\displaystyle\int u^n \ln u\,du = \frac{u^{n+1}\ln u}{n+1} - \frac{u^{n+1}}{(n+1)^2} + C, \qquad n \neq -1.$

1. (8 pts) Compute the *present value* of a continuous annuity that pays at the annual rate $f(t) = 1000t$ for $T = 10$ years, assuming that interest is compounded continuously at the rate $r = 5.2\%$.

2. (8 pts) A firm's marginal revenue function is given by

$$\frac{dr}{dq} = \frac{2q}{\sqrt{4q+25}}.$$

Find the *demand equation* for the firm's product.

3. The monthly demand (Q) for ACME Widgets' product is related to the price per Widget (p) and the average monthly income in the market for their product (Y), by the equation

$$Q = 4\ln(6Y - 5\sqrt{p}),$$

where Q is measured in 1000s of Widgets, the price is measured in dollars and income is measured in thousands of dollars.

a. (4 pts) Compute Q_p and Q_Y when $p = 9$ and $Y = 4$

b. (2 pts) Compute the *price-elasticity of demand* when $p = 10$ and $Y = 3$.

c. (2 pts) Suppose that the firm's price remains fixed but average monthly income in the market for the firm's product increases by \$300. Use your answer to **a.** to estimate the change in demand for ACME's product.

Round your answers to 2 decimal places.

4. A monopolistic firm sells its product in two markets, A and B. The daily demand functions for the firm's product in these markets are

$$Q_A = 150 - 5P_A \text{ and } Q_B = 120 - 3P_B,$$

where Q_A and Q_B are the quantities demanded in markets A and B, respectively, and P_A and P_B are the prices that the firm charges in these two markets. The firm's daly cost function is given by

$$C = 10Q + 400,$$

where Q is the firm's total daily output, which equals the total daily demand for their product (from the two markets).

(a) (8 pts) Find the prices that the firm should charge to maximize their daily profit. Use the second derivative test to verify that the prices you found produce a maximum. What is the firm's max daily profit?

(b) (2 pts) Use the *envelope theorem* and *linear approximation* to estimate the change in the firm's maximum daily profit if their *marginal cost* increases from 10 to 10.75.

5. ACME Widget's (AW) production function is given by

$$Q = 10K^{0.6}L^{0.4},$$

where Q is the firm's annual output, measured in widgets, K is the firm's annual capital input and L is the firm's annual labor input. The price per unit of capital is $p_k = \$8,000$ and the price per unit of labor is $p_l = \$5,000$.

a. (6 pts) Find the levels of capital and labor input that AW should use to ***minimize the cost*** of producing $Q_0 = 10,000$ widgets. What is the minimum cost?

b. (2 pts) What is AW's *marginal cost* at that level of production? Explain your answer.

c. (2 pts) Use the *envelope theorem* and *linear approximation* to estimate the change in AW's (minimum) cost of producing 10,000 widgets, if the price per unit of capital increases to \$8,100 (assuming that all else stays the same).

6. The Jones family's utility function is given by

$$U(x,y,z) = 10\ln x + 15\ln y + 5\ln z,$$

where x, y and z are the quantities of *X-goods, Y-goods* and *Z-goods* that they consume per month. The average prices of these goods are $p_x = \$8$, $p_y = \$20$ and $p_z = \$5$, respectively.

a. (8 pts) Find the quantities of X-goods, Y-goods and Z-goods that the Jones family should consume each month to maximize their utility, given that their monthly XYZ-budget is $B = \$3600$. What is their maximum utility?

b. (2 pts) Use the *envelope theorem* and *linear approximation* to estimate the change in the Jones' monthly utility if the price, p_x, of X-goods *increases* to \$10.50 (assuming that all else stays the same).